

Problems for practice:

<p>Find the derivative:</p> <p>1. <math>y = (x^2 + 3x)^4 \ln(5x + 2)</math></p> <p>2. <math>y = e^{5x+2} \ln(2x - 3)</math></p>	<p>3. When <math>p = 12</math> then <math>q = 600</math>, when <math>p = 20</math> then <math>q = 400</math>.</p> <p>a. Find a function relating <math>p</math> and <math>q</math>.</p> <p>b. Find the corresponding revenue function</p>
<p>4. Given the demand function: <math>q = 1200 - 40p</math>, and a per-item cost of \$5, with an overhead cost of \$200</p> <p>a. find a revenue function</p> <p>b. find a profit function</p> <p>c. figure out how to get the maximum profit. When the profit is maximum:</p> <p>i. What is the production quantity?</p> <p>ii. What is the price?</p> <p>iii. What is the profit?</p>	<p>5. Yearly revenue from 2002 (<math>t=0</math>) to 2008 (<math>t=6</math>) is approximated by :</p> $R = -.16t^3 + 1.4t^2 + 2.1t + 2.4$ (millions of dollars) <p>When is the increase in revenue speeding up? When is it slowing down? When is the revenue increasing most rapidly?</p>
<p>6. Given a cost function of <math>C(x) = 15,000 + 16x + .03x^2</math> (for <math>x</math> items) Find the minimum average cost—tell both the production level (<math>x</math>) and the minimum average cost.</p>	<p>7. Integrate:</p> <p>a. <math>\int 3x^5 + 1.5x^2 - 8x + 3 dx</math></p> <p>b. <math>\int \frac{x}{3} + \frac{2}{x} + \frac{4}{x^2} dx</math></p> <p>c. <math>\int xe^{x^2+3} dx</math></p> <p>d. <math>\int \frac{5}{3x+2} dx</math></p> <p>e. <math>\int (2x+3)\sqrt{x^2+3x+5} dx</math></p>
<p>8. The marginal cost of producing the <math>x</math>th box of widgets is <math>12 + \frac{3}{x^2}</math>. The cost to produce 1 widget is \$500. Find the cost function <math>C(x)</math>. Find the total cost to produce 100 widgets.</p>	

Answers:

$$1. y' = 4(x^2 + 3x)^3 (2x + 3) \ln(5x + 2) + \frac{5(x^2 + 3x)^4}{5x + 2}$$

$$2. y = 5e^{5x+2} \ln(2x-3) + \frac{2e^{5x+2}}{2x-3}$$

3. There are two sets of correct answers:

a. $q = -25p + 900$	a. $p = -.04q + 36$
b. $R = -25p^2 + 900p$	b. $R = -.04q^2 + 36q$

4. There are two sets of correct answers to a and b:

a. $R = 1200p - 40p^2$	a. $R = 30q - .025q^2$
b. $P = 1400p - 40p^2 - 6200$	b. $P = 25q - .025q^2 - 200$

The correct answer to c can be found from either choice of b. The complete answer to c is:

$$P' = 1400 - 80p = 0 \quad P' = 25 - .050q = 0$$

$$p = 17.5 \quad q = 500$$

c. figure out how to get the maximum profit. When the profit is maximum:

- i. 500
- ii. \$17.50
- iii. \$6050

5.

$$R = -.16t^3 + 1.4t^2 + 2.1t + 2.4$$

$$R' = -.48t^2 + 2.8t + 2.1$$

$$R'' = -.96t + 2.8 = 0$$

$$t \approx 2.9$$



rate of increase is speeding up from 0 to 2.9

which is beginning of 2002 to end of 2004

rate of increase is slowing down from 2.9 to 6

which is end of 2004 to beginning of 2008

revenue is increasing most rapidly at 2.9—end of 2004.

$$6. \bar{C}(x) = \frac{15,000 + 16x + .03x^2}{x} = 15,000x^{-1} + 16 + .03x$$

$\bar{C}'(x) = -15,000x^{-2} + .03 = 0$ $\frac{-15000}{x^2} = -.03$ $15000 = .03x^2$ $500,000 = x^2$ $x = 707$	<p>Minimum average cost occurs when making 707 items, and the minimum average cost is \$58.43</p>
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$$7. \text{ a. } \int 3x^5 + 1.5x^2 - 8x + 3 dx = \frac{x^6}{2} + .5x^3 - 4x^2 + 3x + C$$

$$\text{b. } \int \frac{x}{3} + \frac{2}{x} + \frac{4}{x^2} dx = \frac{x^2}{6} + 2 \ln |x| - 4x^{-1} + C$$

$$\text{c. } \int x e^{x^2+3} dx = \frac{1}{2} e^{x^2+3} + C$$

$$\text{d. } \int \frac{5}{3x+2} dx = \frac{5}{3} \ln |3x+2| + C$$

$$\text{e. } \int (2x+3)\sqrt{x^2+3x+5} dx = \frac{2}{3}(x^2+3x+5)^{3/2} + C$$

$$8. C'(x) = 12 + \frac{3}{x^2}$$

$$C(x) = 12x - \frac{3}{x} + C$$

$$C(1) = 12 - 3 + C = 500$$

$$C = 491$$

$$C(x) = 12x - \frac{3}{x} + 491$$

The total cost to produce 100 widgets is \$1690.97